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Stark Effect: Zeeman effect 1897

1913 → Stark demonstrated that every line of the Balmer series of hydrogen, when excited in a strong electric field of at least 100000 volts per cm., is split into a number of components.

Viewed perpendicular to the field;
Some of the components of each line pattern are observed to be plane-polarized with the electric vector parallel to the field \rightarrow P components.

Others polarized with the electric ~~field~~ vector normal to the field (S components).

Viewed parallel to the field only the S component appears \rightarrow unpolarized.

The Stark Effect of Hydrogen

We write the general energy relations for a hydrogen atom in an electric field and then interpret them in terms of atomic models and the observed spectrum lines.

The interaction energy of a hydrogen-like atom in an electric field is given by

$$\Delta E = AP + BP^2 + CP^3 + \dots \quad (1)$$

$\Delta E \rightarrow$ change in the term value of the atom in wave numbers

i.e. the shift is the energy levels from the field-free states to the states in the electric field. (36)

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F → strength of the field in electrostatic units.

Coefficients A, B and C → calculated by classical and quantum mechanical considerations

$$A = \frac{3h}{8\pi^2 m e c} n(n_2 - n_1),$$

$$B = \frac{h^5}{2^{10} \pi^6 m^3 e^6 c} n^4 \left\{ 12n^2 - 3(n_2 - n_1) - 9m_e^2 + 29 \right\}$$

$$C = \frac{3h^9}{2^{15} \pi^{10} m^5 e^{11} c} n^7 \left\{ 23n^2 - (n_2 - n_1)^2 + 11m_e^2 + 35 \right\}$$

— (2)

n → usual quantum number

n_1, n_2, m_e are electron quantum numbers subject to the condition

$$m_e = n - n_2 - n_1 - 1 \quad — (3)$$

The allowed values are

$$n = 1, 2, 3, \dots \infty, \quad n_1 = 0, 1, 2, 3, \dots n-1$$

$$m_e = 0, \pm 1, \pm 2, \dots \pm (n-1), \quad n_2 = 0, 1, 2, 3, \dots n-1$$

If the field is expressed in volts per centimetre the independent constants in these expressions are 6.42×10^5 , 5.22×10^{-16} and 1.53×10^{-25} for A, B, C respectively.

first term is $\epsilon^2 \rightarrow F$ is first power

\rightarrow first order Stark effect

The second term involving F to the second power \rightarrow second order Stark effect etc.

If the field is not too large ($F < 100000$ volts per cm), the lower states of the hydrogen atom (n small) \rightarrow show only a first-order Stark effect.

Such fields result in a symmetrical splitting of the energy levels about their field-free positions.

Second-order effect \rightarrow Always present and becomes large for higher states and higher fields \rightarrow results in a unidirectional displacement of each line.

Weak-field Stark Effect in Hydrogen

Kramers \rightarrow Treatment of the hydrogen atom in a weak electric field \rightarrow neglecting electron spin

Schlapp \rightarrow Treatment including spin

↓
Employs the Dirac electron theory

Weak electric field in hydrogen \rightarrow one in which

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the interaction energy between the electron resultant j^* and the field F is considerably less

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→ In comparison to the magnetic interaction energy between ℓ^* and s^* .

Weak field → Stark splitting is small in comparison to the fine-structure splitting.

In a weak field → electron with spin → classically as a small magnet, does not interact with the field so that the coupling of j^* with F is due only to interaction of ℓ^* with F .

In the classical picture of a precessing atom the electron's mechanical resultant j^* ($h/2\pi$) precesses around the field F . The projections of j^* on the field direction F is given by m_j , $m_j \rightarrow$ taking values differing from each other by unity from $+j$ to $-j$.

Important difference between the Zeeman Effect and the Stark effect → each pair of levels $+m_j$ and $-m_j$ arising from a given level have the same energy when in electric field but different energies when in a magnetic field.

The state $m_j = \frac{3}{2}$, for example has the same energy as that the state $m_j = -\frac{3}{2}$.

Similarly the states $m_j = +\frac{1}{2}$ and $m_j = -\frac{1}{2}$ have the same energy.

Instead of a level $J = \frac{3}{2}$ being split up into four components is the Zeeman effect, there are two levels.

Reason → classical orbital model or the quantum mechanical model of electron clouds. The nature of the forces acting on the electrons are purely electrostatic → energy of the electron in an orbit of given n and l depends only on the inclination of the orbit plane w.r.t. the electric field, or to the distribution of charge in the quantum-mechanical model, → not on the direction of rotation or motion of the electron in its orbit.

States with $+m_j$ and $-m_j$ → correspond to the same inclination of the orbital plane, or the same charge distributions → same g distortion or energy change → due to the applied field. In magnet field the energy depends on the direction of rotation and the energies change sign when m_j changes sign.