

(35)

Stark Effect: Zeeman effect 1897

1913 → Stark demonstrated that every line of the Balmer series of hydrogen, when excited in a strong electric field of at least 100000 volts per cm., is split into a number of components.

Viewed perpendicular to the field;  
Some of the components of each line pattern are observed to be plane-polarized with the electric vector parallel to the field  $\rightarrow$  P components.

Others polarized with the electric ~~field~~ vector normal to the field (S components).

Viewed parallel to the field only the S component appears  $\rightarrow$  unpolarized.

### The Stark Effect of Hydrogen

We write the general energy relations for a hydrogen atom in an electric field and then interpret them in terms of atomic models and the observed spectrum lines.

The interaction energy of a hydrogen-like atom in an electric field is given by

$$\Delta E = AP + BP^2 + CP^3 + \dots \quad (1)$$

$\Delta E \rightarrow$  change in the term value of the atom in wave numbers

i.e. the shift is the energy levels from the field-free states to the states in the electric field. (36)

~~TOHOKU~~

$F \rightarrow$  strength of the field in electrostatic units.

Coefficients A, B and C  $\rightarrow$  calculated by classical and quantum mechanical considerations

$$A = \frac{3h}{8\pi^2 m e c} n(n_2 - n_1),$$

$$B = \frac{h^5}{2^{10} \pi^6 m^3 e^6 c} n^4 \left\{ 12n^2 - 3(n_2 - n_1) - 9m_e^2 + 29 \right\}$$

$$C = \frac{3h^9}{2^{15} \pi^{10} m^5 e^{11} c} n^7 \left\{ 23n^2 - (n_2 - n_1)^2 + 11m_e^2 + 35 \right\}$$

— (2)

$n \rightarrow$  usual quantum number

$n_1, n_2, m_e$  are electron quantum numbers subject to the condition

$$m_e = n - n_2 - n_1 - 1 \quad — (3)$$

The allowed values are

$$n = 1, 2, 3, \dots \infty, \quad n_1 = 0, 1, 2, 3, \dots n-1$$

$$m_e = 0, \pm 1, \pm 2, \dots \pm (n-1), \quad n_2 = 0, 1, 2, 3, \dots n-1$$

If the field is expressed in volts per centimetre the independent constants in these expressions are  $6.42 \times 10^5$ ,  $5.22 \times 10^{-16}$  and  $1.53 \times 10^{-25}$  for A, B, C respectively.

first term is  $\epsilon^2 \rightarrow F$  is first power

$\rightarrow$  first order Stark effect

The second term involving  $F$  to the second power  $\rightarrow$  second order Stark effect etc.

If the field is not too large ( $F < 100000$  volts per cm), the lower states of the hydrogen atom (n small)  $\rightarrow$  show only a first-order Stark effect.

Such fields result in a symmetrical splitting of the energy levels about their field-free positions.

Second-order effect  $\rightarrow$  Always present and becomes large for higher states and higher fields  $\rightarrow$  results in a unidirectional displacement of each line.

Weak-field Stark Effect in Hydrogen

Kramers  $\rightarrow$  Treatment of the hydrogen atom in a weak electric field  $\rightarrow$  neglecting electron spin

Schiff  $\rightarrow$  Treatment including spin

↓  
Employs the Dirac electron theory

Weak electric field in hydrogen  $\rightarrow$  one in which

(38)

the interaction energy between the electron resultant  $j^*$  and the field  $F$  is considerably less

~~TOHOKU~~  
UNIVERSITY

→ In comparison to the magnetic interaction energy between  $\ell^*$  and  $s^*$ .

Weak field → Stark splitting is small in comparison to the fine-structure splitting.

In a weak field → electron with spin → classically as a small magnet, does not interact with the field so that the coupling of  $j^*$  with  $F$  is due only to interaction of  $\ell^*$  with  $F$ .

In the classical picture of a precessing atom the electron's mechanical resultant  $j^*$  ( $h/2\pi$ ) precesses around the field  $F$ . The projections of  $j^*$  on the field direction  $F$  is given by  $m_j$ ,  $m_j \rightarrow$  taking values differing from each other by unity from  $+j$  to  $-j$ .

Important difference between the Zeeman Effect and the Stark effect → each pair of levels  $+m_j$  and  $-m_j$  arising from a given level have the same energy when in electric field but different energies when in a magnetic field.

The state  $m_j = \frac{3}{2}$ , for example has the same energy as that the state  $m_j = -\frac{3}{2}$ .

Similarly the states  $m_j = +\frac{1}{2}$  and  $m_j = -\frac{1}{2}$  have the same energy.

Instead of a level  $J = \frac{3}{2}$  being split up into four components is the Zeeman effect, there are two levels.

Reason → classical orbital model or the quantum mechanical model of electron clouds. The nature of the forces acting on the electrons are purely electrostatic → energy of the electron in an orbit of given  $n$  and  $l$  depends only on the inclination of the orbit plane w.r.t. the electric field, or to the distribution of charge in the quantum-mechanical model, → not on the direction of rotation or motion of the electron in its orbit.

States with  $+m_j$  and  $-m_j$  → correspond to the same inclination of the orbital plane, or the same charge distributions → same g distortion or energy change → due to the applied field. In magnet field the energy depends on the direction of rotation and the energies change sign when  $m_j$  changes sign.