PAPER –II
REAL AND COMPLEX ANALYSIS

TIME: 3 hours **Max. Marks: 100**

UNIT-I

COMPLEX ANALYSIS : Complex numbers : The extended plane and its spherical representation, Analytical functions, Cauchy-Riemann equations, Power series including differentiation and integration within the circle of convergence, Conformal transformation, Linear, Bilinear, Exponential, Trigonometric and Joukowski's transformations. Riemann definition of integration, index of a point with respect to a closed curve and general form of Cauchy's integral formula.

UNIT-II

Simple connectivity, Cauchy's fundamental theorem, Cauchy's integral formula, Liouville's theorem; Morera's theorem, Taylor's theorem, Laurent's theorem; Poisson's integral formula, Maximum Modulus theorem, Rouche's theorem. Singularities, residues, Cauchy's theorem of residues and Evaluation of definite integrals.

UNIT – III

Metric spaces: Examples and properties of a metric space, Open sphere (ball or neighborhood) Open sets, closed sets and the related results, Continuous mappings Cauchy sequence and convergence, complete metric space, Compact spaces and compact sets, Baire's category theorem.

UNIT-1V

Measure Theory: Outer measure of a subset of R'Lebesgue outer measure of a subset of R ,Existence, non-negativity and monotonicity of Lebesgue outer measure, Relation between Lebesgue outer measure and length of an interval, Countable subadditivity of Lebesgue outer measure, translation invariance, Lebesgue outer measure-, (Lebesgue) measurable sets (Lebesgue) measure, Complement, union, intersection and difference of measurable sets, denumerable union and intersection of measurable sets, countable additivity of measure, the class of measurable sets as a algebra, the measure of the intersection of a decreasing sequence of measurable sets, some special classes of measurable sets, intervals, open sets, closed sets, Borel sets, F_6 and G_6 sets. Measurable functions; Different equivalent definition of a measurable function; Scalar multiple, sum,difference and product of measurable functions of measurable function. Measurability of a continuous function and measurability of a continuous image of measurable function.

UNIT-V

Supremum, infimum, limit superior, limit inferior and limit of a sequence of measurable functions. Convergence pointwise and convergence in measures of a sequence of measurable functions. Lebesgue Integral; Characteristic function of a set; simple function; Lebesgue integral of a simple function; Lebesgue integral of a bounded measurable function; Lebesgue integral and Riemann integral of a bounded function defined on a closed interval; Lebesgue integral of a non-negative function; Lebesgue integral of a measurable function; Properties of Lebesgue integral. Convergence Theorems and Lebesgue integral; the bounded convergence theorem; Fatou's Lemma: Monotone convergence theorem; Lebesgue convergence theorem.

Books Recommended:

1. George F-Simmons McGraw Hill Book Co. : Introduction to Topology and Modem Analysis,
2. S.I.Hu : Elements of Real Analysis
3. H.L.Royden : Real Analysis.
4. G.N.Purohit : Lebesgue Measure and Integration.
5. E.G.Phillips : Functions of a complex variable.
6. E.T.Copson : An introduction to the Theory of functions of a Complex variable.