

PAPER-V

NUMBER THEORY

TIME: 3 hours

Max. Marks: 100

UNIT-I

Divisibility: Gcd and Lcm of two or more integers, Euclidean, algorithm, the linear diophantine equation $ax + by = c$. Prime Numbers, composite numbers, infinitude of primes, fundamental theorem of arithmetic.

Congruences: Basic properties, divisibility tests, linear congruences. Application of Congruences: Fermat's little theorem, Euler's generalization, Wilson's theorem Chinese remainder theorem.

UNIT-II

Number Theoretic functions: T.J, and, Multiplicative functions, Mobius inversion formula, the greatest integer function. Primitive Roots and Indices, Primitive roots, characterization of natural numbers having primitive roots, theory of indices, solution of certain congruence, through indices. Quadratic Residues: Quadratic residues and quadratic non residues of an integer in general and of a prime in particular, Gauss Lemma and its applications, the quadratic reciprocity law.

UNIT-III

Special Numbers: Fibonacci numbers, Fermat's numbers, Perfect numbers. Diophantine Equations: Representation of integers as sums of 2,3 and 4 squares.

Continued Fractions: Finite and infinite continued fraction convergent of a given continued fraction and their properties, Uniqueness of a continued fraction Periodic continued fraction, Pell's equation in general, characterization of solutions of $x-dy = 1$ in terms of its smallest positive solution.

UNIT-IV

Algebraic number fields and their rings of integers, Calculations for quadratic and cubic cases. Localization, Galois extension.

UNIT-V

Dedekind rings, discrete valuation rings completion, unramified and ramified extensions, different discriminates, cyclotomic fields, roots of unity.

Books Recommended:

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| 1. Donald M. Burton | : | Elementary Number Theory, Allyn and Bacon Inc. |
| 2. Niven & H.S. Zuckerman | : | An Introduction to the Theory of Numbers. Willey eastern India Ltd. |
| 3. Lang, S. | : | Algebraic Number theory, GTM Vol. 110, Springer-Verlag 194. |